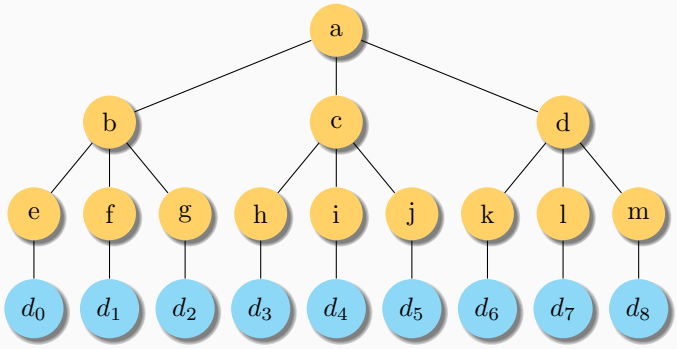
Exercise 1 (Non-Binary Merkle Trees)

We have seen binary merkle trees, where each non-leaf node has two children. A *ternary merkle tree* is a merkle tree where each non-leaf node has three children.

1. Draw a balanced ternary merkle tree with 9 leaves.



2. Give the merkle proof for the first leaf.

[(f,g), (c,d)]

Exercise 2 (Efficiency of Binary Merkle Trees)

Binary merkle trees are efficient: to prove you correctly delivered one of n files you need to deliver a proof of size $\log_2(n)$.

How about ternary merkle trees? Are they more or less efficient than binary merkle trees? How much more/less efficient are they?

- Proof size for a binary merkle tree with n leaves:

$$\log_2(n)$$

- Proof size for a ternary merkle tree with n leaves:

$$\begin{aligned}\log_3(n) \cdot 2 &= (\log_2(n)/\log_2(3)) \cdot 2 \\ &= (\log_2(n)/(\ln(3)/\ln(2))) \cdot 2 \\ &\approx \log_2(n) \cdot 1.26\end{aligned}$$

Converting base of logarithm:

$$\log_b(n) = \log_x(n)/\log_x(b)$$